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# LARGE-SCALE VORTICITY GENERATION DUE TO DISSIPATING WAVES IN THE SURF ZONE

Philippe Bonneton, Nicolas Bruneau and Bruno Castelle

Université Bordeaux 1, CNRS, UMR 5805-EPOC
avenue des Facultés, Talence, F-33405, France

Fabien Marche

Université Montpellier 2, Institut de Mathématiques et de Modélisation de Montpellier
CC 051, Place Eugene Bataillon, 34095 Montpellier cedex 5, France

Abstract. In this paper, we investigate the mechanisms which control the generation of wave-induced mean current vorticity in the surf zone. From the vertically-integrated and time-averaged momentum equations given recently by Smith [21], we obtain a vorticity forcing term related to differential broken-wave energy dissipation. Then, we derive a new equation for the mean current vorticity, from the nonlinear shallow water shock-wave theory. Both approaches are consistent, under the shallow water assumption, but the later gives explicitly the generation term of vorticity, without any ad-hoc parametrization of the broken-wave energy dissipation.

1. **Introduction.** In the nearshore, alongshore variations in waves and wave-induced currents are ubiquitous. These variations can be due to alongshore inhomogeneities in the incident wave field or in the local bathymetry. As shown theoretically by Peregrine [18], non-uniformities along the breaking-wave crest drive vertical vorticity. The vorticity that is being discussed here is not the small scale vorticity caused directly by wave breaking and subsequent turbulent motions, but the vorticity in the form of quasi two-dimensional eddies (usually called 'macrovortices') with horizontal scales larger than the local water depth. The most frequently observed nearshore macrovortices are rip current circulations. Rip currents are shore-normal, narrow, seaward-flowing intense currents that originate within surf zone, extend seaward of the breaking region, and are associated with horizontal eddies. These macrovortices play a major role in circulation and mixing processes in the nearshore.

Studies described in Peregrine [18] and Brocchini et al. [4] draw attention to the way in which non-uniformities along the bore crests lead to generation of vertical vorticity. They proceeded to a direct analysis of vorticity, at the wave's time scale, modeling the breaking event as the development of a surface and current discontinuity in the non-linear shallow water equations (also called Saint Venant equations). These breaking wave processes induce wave-averaged current and mean vorticity. The aim of the present paper is to investigate wave-averaged mean flow vorticity due to differential wave breaking in the surf zone. Mean vertical vorticity equations are

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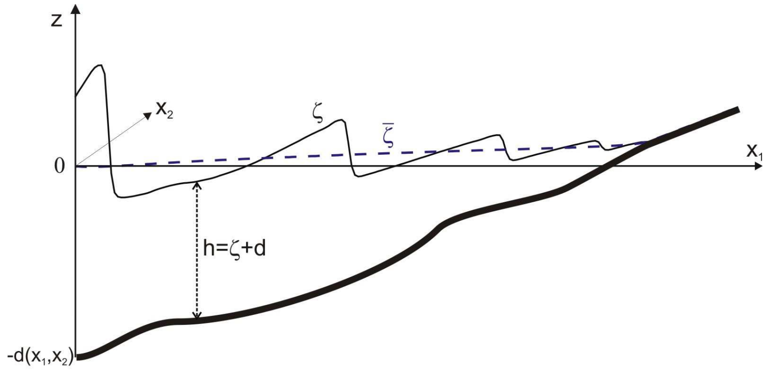


FIGURE 1. Definition sketch for the surf zone.

derived from both the vertically-integrated and time-averaged momentum equation given by Smith [21] and the nonlinear shallow water shock wave theory.

## 2. **Wave-averaged model.**

The nearshore circulation is generally determined by the depth-integrated and time-averaged equations of mass and momentum (see Phillips [19] ). In this section, we analyze the mechanism of vertical vorticity generation in the framework of this classical approach.

### 2.1. **Mass and momentum conservation equations.**

The vertically-integrated mass and horizontal momentum budgets are examined. For the convenience of vertical integration, the vertical coordinate $z$ is treated separately from the horizontal ones $(x_1, x_2)$. As shown in figure 1, the fluid is bounded between the bed, $z = -d(x_1, x_2)$, and the free surface elevation, $z = \zeta(x_1, x_2, t)$. For simplicity, we consider periodic waves of period $T$ and, in this section, turbulence is neglected. Assuming a time scale separation between waves and currents, the horizontal flow velocity, $v_i(x_1, x_2, z, t)$ can be separated into mean, $\bar{v}_i$, and wave, $\tilde{v}_i = v_i - \bar{v}_i$, components, where the time operator $\overline{(.)}$ is defined as: $\overline{(.)} = \frac{1}{T}\int_t^{t+T} (.)\, d\tau$.

Vertical integration of the mass equation, combined with kinematics boundary conditions and subsequent time integration result in

$$\frac{\partial \bar{h}}{\partial t} + \frac{\partial \bar{M}_j}{\partial x_j} = 0 \ , \tag{1}$$

where $\bar{h}$ is the mean water depth and $\bar{M}_i = \overline{\int_{-d}^{\zeta} v_i \, dz}$ is the total horizontal momentum.

Assuming that the mean horizontal velocity $U_i = \bar{v}_i$ is depth-uniform and that the mean pressure is hydrostatic, the horizontal momentum equations can be written as (see Phillips [19] )

$$\frac{\partial \bar{M}_i}{\partial t} + \frac{\partial}{\partial x_j}\left((\bar{M}_i \bar{M}_j)/\bar{h}\right) + g\bar{h}\frac{\partial \bar{\zeta}}{\partial x_i} = -\frac{\partial S_{ij}}{\partial x_j} \ , \tag{2}$$

where $g$ is the gravitational acceleration, $\rho$ the fluid density and $S_{ij}$ is the radiation stress which can be expressed as

$$S_{ij} = \overline{\int_{-d}^{\zeta} (\frac{P}{\rho}\delta_{ij} + \tilde{u}_i\tilde{u}_j)\ dz} - \frac{1}{2}g\bar{h}^2\delta_{ij} - \frac{\tilde{M}_i\tilde{M}_j}{\bar{h}}\ , \tag{3}$$

with $P$ the pressure. The radiation stress represents the excess momentum flux that results from wave motions. The classical approach, based on equations (1) and (2), has been used in many nearshore applications. However, the radiation stress encompasses different wave processes. In particular, the radiation stress gradient combines non-dissipative wave effects as well as dissipative effects due to wave breaking, which alone can create mean flow vorticity.

Recently, Smith [21] presented a reformulation of this theory. The total momentum, $\bar{M}_i = \bar{h}U_i + \overline{\int_{\bar{\zeta}}^{\zeta} \tilde{v}_i\ dz}$, is splited into mean current momentum, $\bar{h}U_i$, and wave momentum, $\tilde{M}_i = \overline{\int_{\bar{\zeta}}^{\zeta} \tilde{v}_i\ dz}$. By subtracting the waves' momentum budget (based on linear wave theory and an ad-hoc parametrization of the broken-wave energy dissipation) from the total, Smith [21] obtained a new set of equations

$$\frac{\partial\bar{h}}{\partial t} + \frac{\partial\bar{h}U_j}{\partial x_j} = -\frac{\partial\tilde{M}_j}{\partial x_j} \tag{4}$$

$$\frac{\partial\bar{h}U_i}{\partial t} + \frac{\partial}{\partial x_j}\left(\bar{h}U_iU_j\right) + g\bar{h}\frac{\partial\bar{\zeta}}{\partial x_i} = \tilde{F}_i\ , \tag{5}$$

where the wave force $\tilde{F}_i$ acting on the mean flow can be written as

$$\tilde{F}_i = \frac{k_i D_{b_m}}{\sigma} + \tilde{M}_j(\frac{\partial U_j}{\partial x_i} - \frac{\partial U_i}{\partial x_j}) - U_i\frac{\partial\tilde{M}_j}{\partial x_j} - \bar{h}\frac{\partial\tilde{J}}{\partial x_i}\ ,$$

with, $\sigma$ the intrinsic wave frequency, $k_i$ the wave vector, $D_{b_m}$ the broken-wave energy dissipation, $E$ the wave energy and $\tilde{J} = E\frac{k}{\sinh(2k\bar{h})}$. The broken-wave energy dissipation $D_{b_m}$ is generally estimated from the analogy between a breaking-wave and a hydraulic jump (see Thornton et Guza [20]) and can be expressed as

$$D_{b_m} = \frac{g}{4T}\frac{H^3}{\bar{h}}\ , \tag{6}$$

where $H$ is the wave height. The first term in the wave force expression is associated with the dissipation of wave momentum due to wave breaking. This loss of wave momentum is directly transfered to the mean flow. We will see in the next section that this dissipative term controls the generation of mean current vorticity. The Smith's model is equivalent to the system (1) and (2), but allows a better understanding of the exchanges between wave momentum and mean current momentum.

2.2. **Vorticity equation for the mean current.** Bühler [7] presented a general theoretical analysis of wave-driven currents and vortex dynamics due to dissipating waves. He identified a dissipative force within the radiation-stress convergence, which controls the mean-current vorticity generation. In this section, we use the non-conservative form of the mean horizontal momentum equation (5) to explicitly state the dissipative force due to wave-breaking in the nearshore, and then we derive the mean current vorticity equation.

Equation (5) can be rearranged in a non-conservative form

$$\frac{\partial\mathbf{U}}{\partial t} + (\mathbf{U}.\nabla)\mathbf{U} + g\nabla\bar{\zeta} = \tilde{\mathbf{G}}\ , \tag{7}$$

where the wave force $\tilde{\mathbf{G}}$ acting on the mean flow can be written as

$$\tilde{\mathbf{G}} = D\mathbf{e}_k + \frac{\tilde{\mathbf{M}}}{\bar{h}} \wedge (\nabla \wedge \mathbf{U}) - \nabla \tilde{J} + \mathbf{T_u} \ ,$$

with $\mathbf{e}_k = \mathbf{k}/\|\mathbf{k}\|$. The dissipative force $D$ is given by $D = \frac{D_{b_m}}{\bar{h} c_\phi}$ and, using (6), can be written

$$D = \frac{g}{4 c_\phi T} \frac{H^3}{\bar{h}^2} \ , \tag{8}$$

where $c_\phi$ is the norm of the phase velocity. The last term, $T_{u_i} = \nu_t \frac{\partial}{\partial x_j}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$, is added to parametrize the turbulent momentum diffusivity of the mean current. For simplicity, we consider in this paper a constant eddy viscosity $\nu_t$. Note that this choice of parameterization is not energetically consistent (see Gent [10]). A better choice, regarding consistency, would be the viscous formulation proposed by Marche [16], which was asymptotically derived from the 3D Navier stokes equations with free surface. However, the present derivation of an autonomous vorticity equation can not be extended to such a formulation.

The equation for the mean flow vertical vorticity, $\omega = \frac{\partial U_2}{\partial x_1} - \frac{\partial U_1}{\partial x_2}$, is obtained straightforward by taking the curl of equation (7), which yields

$$\frac{\partial \omega}{\partial t} + \nabla . (\omega \mathbf{U}_T) = \nu_t \nabla^2 \omega + \nabla \wedge D\mathbf{e}_k \ , \tag{9}$$

where $\mathbf{U}_T = \frac{\bar{\mathbf{M}}}{\bar{h}} = \mathbf{U} + \frac{\tilde{\mathbf{M}}}{\bar{h}}$ is the mean transport velocity and the cross product is treated as a scalar. In equation (9), the first term on the right-hand side expresses diffusion of the mean current vorticity and the second one is a vorticity production term, which can be approximated by $\nabla D \wedge \mathbf{e}_k$. This term is active in presence of dissipative waves, when the gradient of $D$ is not parallel to the wave vector. This result is in agreement with the study of Peregrine [18], who showed that vertical vorticity generation is associated with non-uniformities in bores.

The equation (9) provides a simple and efficient model to understand the generation of vortical motions in the surf zone, such as longshore currents or rip currents. Explanation of rip current generation following the classical radiation stress approach (see Castelle and Bonneton [8] or MacMahan et al. [14] ) is difficult because a large part of the wave driving force (gradients in the radiation stress) does not generate currents as it is irrotational. In our approach, the rotational part of the wave driving force is clearly identified, which allows a better explanation of wave-induced vortical rip currents.

A qualitative explanation of rip current dynamics is as follow. Due to refraction, wave breaking is more intense over the shoal. This differential wave breaking (see figure 2) induces a circulation with shoreward currents over the shoals and a seaward current (called rip current) over the lower part of the bathymetry. To describe more qualitatively this phenomenon, we present in figure 3 a numerical simulation of wave-induced currents and vorticity over a transverse bar and rip morphology which is typically observed on the aquitanian coast (see Castelle and Bonneton [8]). The computations were performed with a numerical model coupling the spectral wave code SWAN (Booij et al. [3]) with the flow model MARS 2DH, which solves the equations (4) and (7) (see Bruneau et al. [5] for more details). We observe in figure 3a that the wave-induced vorticity term, $\nabla D \wedge \mathbf{e}_k$, is intense close to the rip channel, with a clockwise forcing in the upper part and an anti-clockwise forcing in the lower part. This is due to strong alongshore variations in the bathymetry close

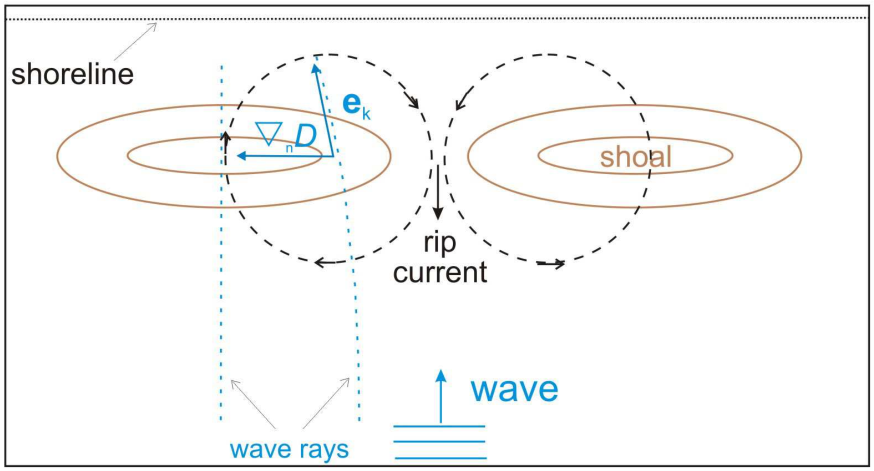


FIGURE 2. Schematic representation of rip circulation induced by wave breaking over shoals.

to the rip channel. This wave forcing generates two main circulation cells on both sides of the channel (see 3b), which are associated with shoreward currents over the bars and a seaward current in the rip channel. The mean vorticity field (figure 3b) is strongly correlated to the vorticity forcing term (figure 3a).

3. **Shock-wave model.** We showed in the previous section that the dissipative force $D$ plays a key role in the surf zone circulation. However, this force has been introduced in the Smith's theory in an ad-hoc way, by adding a dissipative breaking term in the wave-action conservation equation. In this section, we show that we can explicitly derive such a dissipative force, from the shock-wave theory for Saint Venant (SV) equations, without any ad-hoc parametrization. Indeed, SV equations are a good approximation to wave motion in the surf zone (see Hibbert and Peregrine [12], Kobayashi et al. [13] or Bonneton [2]).

3.1. **1D cross-shore mean flow equations.** The one-dimensional SV equations are given by

$$\frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x_1} = 0 \tag{10}$$

$$\frac{\partial hu}{\partial t} + \frac{\partial}{\partial x_1}\left(hu^2 + \frac{1}{2}gh^2\right) = gh\frac{\partial d}{\partial x_1}\ , \tag{11}$$

where $u(x_1,t) = \frac{1}{h}\int_{-d}^{\zeta} v_1\, dz$ is the depth-averaged cross-shore velocity.

Following the concept of "weak solutions" (Godlewski and Raviart [11], Whitham [22]), we can approximate the broken-wave solution (figure 4a) by introducing a discontinuity (see figure 4b) satisfying jump conditions based on mass and momentum conservation across the shock:

$$\begin{aligned} -c_b[h] + [hu] &= 0 \\ -c_b[hu] + [hu^2 + \frac{1}{2}gh^2] &= 0 \quad , \end{aligned}$$

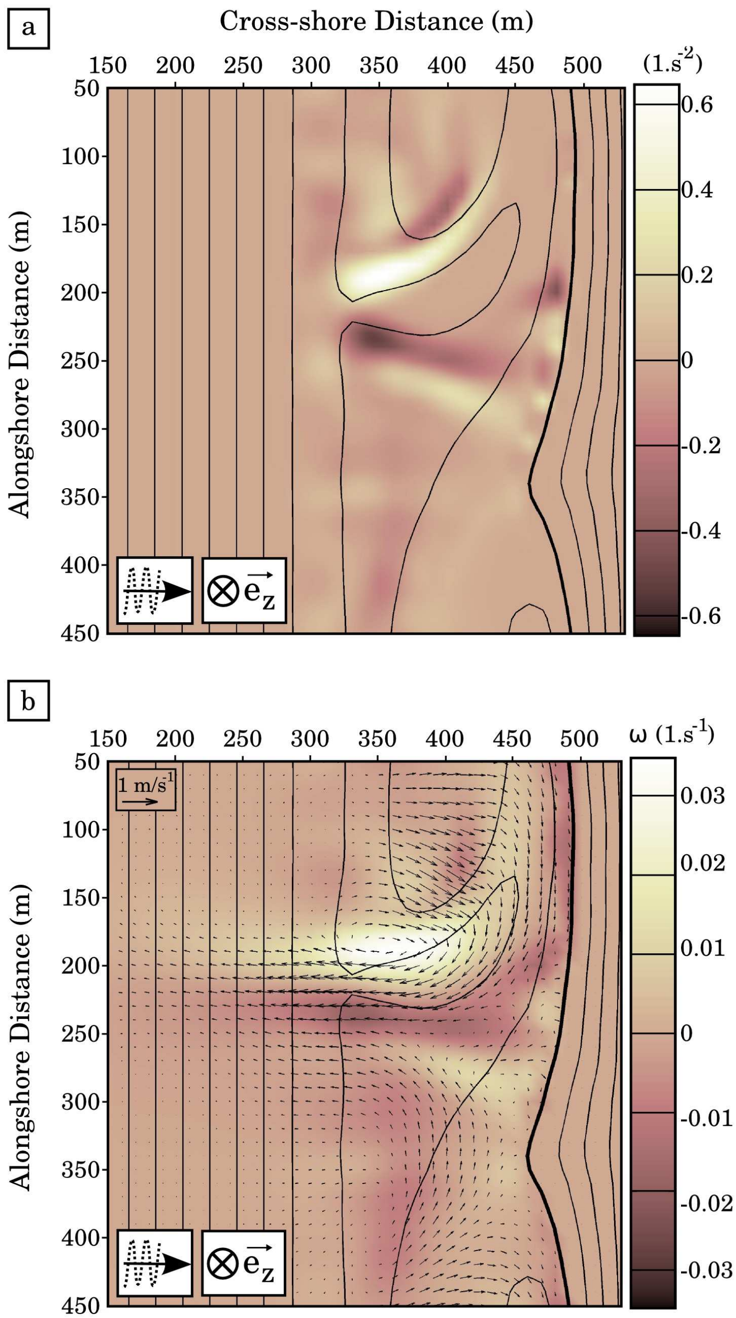


FIGURE 3. Numerical simulation of wave-induced circulation over a transverse bar and rip system. Offshore waves: $H_s = 1.5$ m and $T = 9$ s; thin lines: isobaths between -12 m (offshore) and 0 m (landward); bold line: shoreline. (a) Vorticity forcing term $\nabla D \wedge \mathbf{e}_k$; (b) vorticity field with superimposed mean transport velocity vector field $\mathbf{U}_T$.

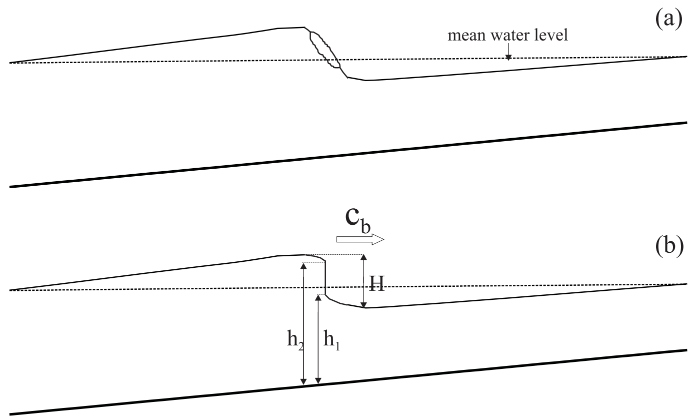


FIGURE 4. Definition sketch. (a) Cross-section of a broken-wave in the surf zone; (b) shock representation. $c_b$ is the broken-wave celerity, $H$ the wave height, $h$ the water depth and subscript 1 and 2 indicate values respectively ahead and behind the shock.

where the brackets [ ] indicate a jump in the quantity and $c_b$ is the shock velocity. A conventional notation is to use subscript 1 and 2 for values ahead and behind the shock respectively (see figure 4b). So the jump conditions can also be written in the form

$$u_1 - c_b = -\left(\frac{gh_2}{2h_1}(h_2 + h_1)\right)^{\frac{1}{2}} \tag{12}$$

$$u_2 - c_b = -\left(\frac{gh_1}{2h_2}(h_2 + h_1)\right)^{\frac{1}{2}} . \tag{13}$$

Mathematically, the composite solution, composed of continuously differentiable parts satisfying equations (10) and (11), together with jump conditions (12) (13), can be considered as a weak solution of the SV equations.

Like in the preceding section, the flow is separated into mean and wave components: $u = \bar{u} + \tilde{u}$. Time averaging the conservative mass equation gives:

$$\frac{\partial \bar{h}}{\partial t} + \frac{\partial \bar{h}\bar{u}}{\partial x_1} = -\frac{\partial \tilde{M}}{\partial x_1} , \tag{14}$$

where $\tilde{M} = \overline{\tilde{\zeta}\tilde{u}}$ is the wave momentum.

To derive the mean current momentum equation we develop the expression of the gradient $\frac{\partial \bar{M}}{\partial x_1}$, where $M = \frac{1}{2}u^2 + g\zeta$,

$$\begin{aligned} T\frac{\partial \bar{M}}{\partial x_1} &= \frac{\partial}{\partial x_1}\left(\int_t^{t+T} M\, d\tau\right) = \frac{\partial}{\partial x_1}\left(\int_t^{t_s^-} M\, d\tau\right) + \frac{\partial}{\partial x_1}\left(\int_{t_s^+}^{t+T} M\, d\tau\right) \\ &= \int_t^{t_s^-} \frac{\partial M}{\partial x_1}\, d\tau + \frac{dt_s}{dx_1}M(t_s^-) + \int_{t_s^+}^{t+T} \frac{\partial M}{\partial x_1}\, d\tau - \frac{dt_s}{dx_1}M(t_s^+) , \end{aligned}$$

where $t_s(x_1)$ is the time at which the wave front (or the shock) is located in $x_1$. In continuous parts of the flow, the momentum equation (11) is equivalent to the following equation:

$$\frac{\partial u}{\partial t} + \frac{\partial M}{\partial x_1} = 0 \; . \qquad (15)$$

Inside intervals $[t, t_s^-]$ and $[t_s^+, t+T]$ the wave solution is continuous and so $\frac{\partial M}{\partial x_1}$ can be evaluated from (15), which yields

$$T\frac{\partial \bar{M}}{\partial x_1} = -\int_t^{t_s^-} \frac{\partial u}{\partial t}\, d\tau - \int_{t_s^+}^{t+T} \frac{\partial u}{\partial t}\, d\tau + \frac{1}{c_b}[M] = -T\frac{\partial \bar{u}}{\partial t} + \frac{1}{c_b}([M] - c_b[u]) \; , \quad (16)$$

and finally

$$\frac{\partial \bar{u}}{\partial t} + \bar{u}\frac{\partial \bar{u}}{\partial x_1} + g\frac{\partial \bar{\zeta}}{\partial x_1} = D - \frac{\partial \tilde{J}}{\partial x_1} \; , \qquad (17)$$

where $\tilde{J} = \frac{1}{2}\overline{\tilde{u}^2}$ and $D = \frac{1}{c_b T}([M] - c_b[u])$. $D$ is determined by using shock conditions (12) and (13) and writes

$$D = \frac{g}{4c_b T}\frac{(h_2 - h_1)^3}{h_2 h_1} \; . \qquad (18)$$

where $H = h_2 - h_1$ is the wave height.

Equations (14) and (17) are consistent, under the shallow water assumption, with the system (4, 7) derived in section 2. In equation (17), the dissipative force $D$ is now explicitly derived from the shock wave approach. It can be applied either to saturated breakers ($H = h_2 - h_1$), as in classical approaches (see Thornton et Guza [20] and Bonneton [1]), and to non-saturated breakers ($h_2 - h_1 < H$, see figure 4b). We can see that equation (8) is an approximation of equation (18) limited to saturated breakers and based on estimating the broken wave celerity $c_b$ by the linear phase velocity $c_\phi$, which is a crude estimate of $c_b$ (see Bonneton [1]).

For stationary cross-shore mean flows we can obtain a new equation for wave-induced mean water level increase (wave setup)

$$g\frac{\partial \bar{\zeta}}{\partial x_1} = D - \frac{\partial}{\partial x_1}\left(\tilde{J} + \frac{1}{2}\frac{\tilde{M}^2}{\bar{h}^2}\right) \; . \qquad (19)$$

The main contribution for the wave setup is due to the dissipative force $D$. Equation (19) is interesting from a physical point of view because, conversely to the classical theory based on radiation stresses (Phillips [19]), this equation provides a simple and explicit relation between wave setup and energy dissipation. Bonneton [2] showed that equation (19) can represent an alternative to the classical radiation stress method for computing wave setup in the surf zone.

3.2. **2D mean flow equations.** To extend the previous approach to a two-dimensional one, we consider the SV equations in a curvilinear coordinate system based on rays and their orthogonals. The rays are the orthogonal trajectories of the successive positions of the wavefront. The velocity field, $\mathbf{u} = (u_1, u_2)$, writes $\mathbf{u} = V\mathbf{e}_k$. Time averaging of the two-dimensional SV equations gives

$$\begin{aligned} \frac{\partial \bar{h}}{\partial t} + \nabla.(\bar{h}\bar{\mathbf{u}}) &= -\nabla.\tilde{\mathbf{M}} && (20) \\ \frac{\partial \bar{\mathbf{u}}}{\partial t} + (\bar{\mathbf{u}}.\nabla)\bar{\mathbf{u}} + g\nabla\bar{\zeta} &= D\mathbf{e}_k - \nabla\tilde{J} - \overline{\tilde{\omega}(\tilde{\mathbf{u}} \wedge \mathbf{e}_z)} \; , && (21) \end{aligned}$$

where $\tilde{\omega}$ is the wave component of the vertical vorticity and $\tilde{J} = 0.5\overline{\tilde{V}^2} = 0.5(\overline{\tilde{u}_1^2} + \overline{\tilde{u}_2^2})$.

From the non-conservative momentum equation (21) it is straightforward to get the equation for the mean flow vorticity

$$\frac{\partial \omega}{\partial t} + \nabla . \left( \bar{\omega} \bar{\mathbf{u}} + \overline{\tilde{\omega} \tilde{\mathbf{u}}} \right) \quad = \quad \nabla \wedge \left( D \mathbf{e}_k \right) \, . \tag{22}$$

Equation (22) is consistent, under the shallow water assumption, with equation (9), with in particular the same vorticity forcing term, $\nabla \wedge (D\mathbf{e}_k)$, related to differential wave dissipation. However, in the present approach the dissipative force $D$ is explicitly given by the nonlinear shock-wave theory.

4. **Conclusion.** In this paper, we have investigated the mechanisms which control the generation of wave-induced mean current vorticity in the surf zone. From the vertically-integrated and time-averaged momentum equations given recently by Smith [21], we obtained a vorticity forcing term related to differential broken-wave energy dissipation. Then, we derived a new equation for the mean current vorticity, from the nonlinear shallow water shock-wave theory. Both approaches are consistent, under the shallow water assumption, but the later gives explicitly the generation term of vorticity, without any linear assumption and ad-hoc parametrization of the broken-wave energy dissipation. Further work is required to evaluate the predictive capability of the shock-wave approach in comparison with recent large-scale laboratory experiments (Castelle et al. [9]) and field measurements (Bruneau et al. [6]). In these experiments, macrovortices are mainly generated by wave-bathymetry interactions above strongly varying bathymetry. The numerical simulation of such topographically controlled macrovortices requires the use of high order robust well-balanced schemes (Marche et al.[15, 17]). This is currently under investigation.

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Received April 2009; revised May 2009.

*E-mail address*: `p.bonneton@epoc.u-bordeaux1.fr`
*E-mail address*: `nbruneau@lnec.pt`
*E-mail address*: `b.castelle@epoc.u-bordeaux1.fr`
*E-mail address*: `fmarche@math.univ-montp2.fr`